Letters

Comments on "Nonuniform Layer Model of a Millimeter-Wave Phase Shifter"

KAZUHIKO OGUSU AND IKUO TANAKA

In the above paper,[1] Butler *et al.* have investigated theoretically the propagation characteristics of the dielectric waveguide with plasma layer created by an exponentially absorbed optical beam. They have solved numerically the complex wave equation by using a multipoint boundary-value differential equation solver. On the other hand, we have already treated the same nonuniform layer model by using a multilayer staircase method [2]. In this approach, the actual permittivity profile of the waveguide is approximated by the finite number of steps. The wave equation is solved for each step and the complex propagation constant is determined so as to satisfy the boundary conditions at all interfaces. The mathematical formulation is very simple and an accurate solution can be obtained by increasing the number of steps M. The details of the method can be found in [3] and [4].

In this letter, we would like to present the numerical results calculated by using the multilayer staircase method and examine the validity of results given by Butler *et al.* [1]. Fig. 1 shows the propagation characteristics of the lowest order TM mode in a silicon waveguide for various plasma decay constants W_d. This numerical example is the same as that in Fig. 10 of [1]. Our exact values (corresponding to $M = \infty$) of the complex propagation constant were determined by extrapolating the plots as a function of $1/M$. The material parameters used in our calculations are the same as those used by Lee *et al.* [5]. The results given by Butler *et al.* were reproduced from the figure in their paper [1]. Our numerical results are in good agreement with their results. However, there is a small discrepancy for large plasma density ($\gtrsim 10^{17}$ cm^{-3}), as shown in Fig. 1. The reason for the discrepancy has not been made clear at the present stage. For this comparison, it should be noted that their attenuation values shown in Fig. 1 are double the original values in their paper. This discrepancy seems to be due to an incorrect definition of decibels. The attenuation in dB/cm is defined as $-10\log(e^{-2\alpha}) = 8.686\alpha$, where α is the attenuation constant with the dimension of 1/cm [6]. Although Butler *et al.* have described the agreement with results given by Lee *et al.* [5] as good, note that there is also the same mistake in [5] (see eq. (44)).

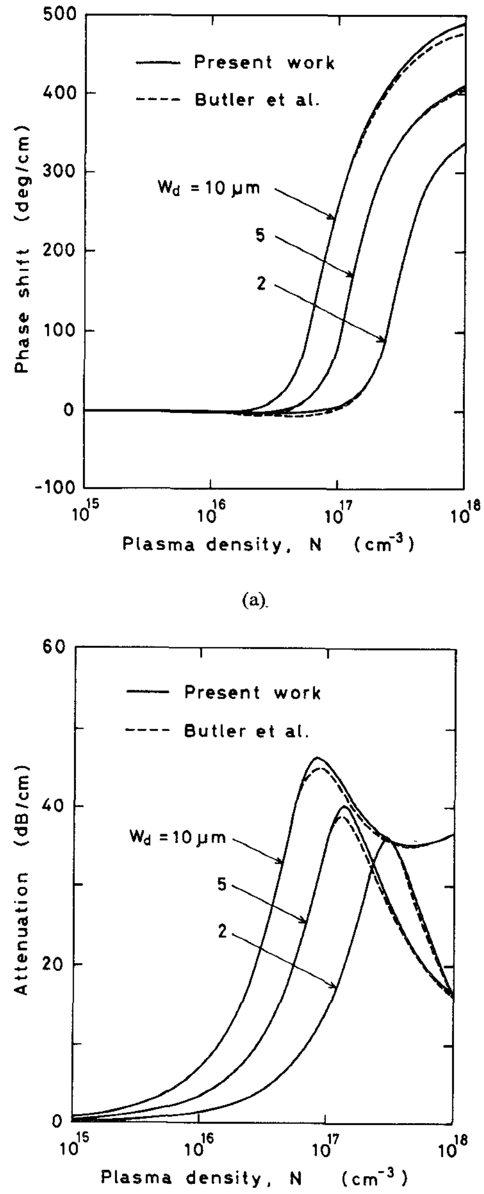

Fig. 1. Propagation characteristics of the lowest order TM mode in a silicon waveguide of 1-mm thickness at 94 GHz. This numerical example refers to Fig. 10 of [1]. (a) Phase shift. (b) Attenuation.

Reply[2] by J. K. Butler, T. F. Wu, and M. W. Scott[3]

Ogusu and Tanaka correctly state that the definition of decibel as used in our paper [1] and in [5] is not the standard definition. As they point out, (44) in [5] defines the decibel to be one-half the value used by Ogusu and Tanaka. The reason is given in the sentence before (44), where the authors state that they are plotting wave attenuation and not power attenuation. We adopted the same procedure so that our results could be directly compared with the results of [5]. However, we agree that the definition we used is not standard, which we should have explicitly stated.

We were not aware that the staircase method had been used previously to compute phase shift and attenuation in a nonuniform waveguide in [2]. It appears that the staircase method can be used to produce relatively accurate results depending upon the

Manuscript received March 10, 1986.
K. Ogusu is with the Faculty of Engineering, Shizuoka University, Hamamatsu, 432 Japan.
I. Tanaka is with the College of Engineering, Shizuoka University, Hamamatsu, 432 Japan.
IEEE Log Number 8610278.
[1] J. K. Butler, T. F. Wu, and M. W. Scott, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 147–155, Jan. 1986.
[2] Manuscript received May 14, 1986.
[3] J. K. Butler and T. F. Wu are with Southern Methodist University, Dallas, TX 75275.
M. W. Scott is with the LTV Aerospace and Defense Company, Dallas, TX 75265-003.
IEEE Log Number 8611355.

number of partitions. The major discrepancy between our computations and those of Ogusu and Tanaka appears to be around carrier densities of 10^{17} cm^{-3}, which is near maximum wave attenuation. We note that the two methods have been compared by other researchers [7], [8] in regard to accuracy and efficiency. Nevertheless, we feel that the simultaneous solution of the eigenvalues and differential equations can be more effectively performed by the multipoint boundary-value solver as described in our paper.

References

[1] J. K. Butler, T. F. Wu, and M. W. Scott, "Nonuniform layer model of a millimeter-wave phase shifter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 147–155, Jan. 1986.

[2] K. Ogusu, I. Tanaka, and H. Itoh, "Propagation properties of dielectric waveguides with optically induced plasma layers," *Trans. IECE Japan*, vol. J66-C, pp. 39–46, Jan. 1983 (Transl.: *Electronics and Communications in Japan*, vol. 66, pp. 95–102, Jan. 1983).

[3] Y. Suematsu and K. Furuya, "Characteristic modes and scattering loss of asymmetric slab optical waveguides," *Trans. IECE Japan*, vol. J56-C, pp. 277–284, May 1973.

[4] M. J. Adams, *An Introduction to Optical Waveguides*. New York: Wiley, 1981, sect. 5.4.

[5] C. H. Lee, P. S. Mak, and A. P. DeFonzo, "Optical control of millimeter-wave propagation in dielectric waveguides," *IEEE J. Quantum Electron.*, vol. QE-16, pp. 277–288, Mar. 1980.

[6] S. Ramo, J. R. Whinnery, and T. Van Duzer, *Fields and Waves in Communication Electronics*. New York: Wiley, 1965, p. 423.

[7] J. M. Arnold, G. A. E. Crone, and P. J. B. Clarricoats, "Comparison of numerical computations of optical-waveguide transmission parameters," *Electron. Lett.*, vol. 13, pp. 273–274, May 12, 1977.

[8] J. M. Arnold, "Stratification methods in the numerical analysis of optical-waveguide transmission parameters," *Electron. Lett.*, vol. 13, pp. 660–661, Oct. 27, 1977.

Comments on "Self-Adjoint Vector Variational Formulation for Lossy Anisotropic Dielectric Waveguide"

ROLAND HOFFMANN

In the above paper,[1] the authors present a "new variational formula" and its derivation. A careful inspection of the text shows that there are a number of errors and wrong conclusions with the fatal consequence that the final variational formula [1, eq. (37)] is incorrect. The main fallacy of the authors appears to be the derivation of the adjoint solution, and the following discussion will be restricted to this point.

The authors state correctly [1, eq. (11)] that, for real inner product, the eigenvalue of the adjoint problem is $\gamma^a = -\gamma$ (while [1, eq. (12)] should read $\gamma^a = -\gamma^*$). The arguments following this equation are not complete and the conclusions are not clear. It is in fact true that there are several classes of waveguides with the property that γ as well as $-\gamma$ is a valid eigenvalue of the problem. But in contrast to the authors method, this may appear as a solution of [1, eq. (1)] as well as [1, eq. (2)] by taking into account that the electromagnetic fields, i.e., the eigenvectors are different for $+\gamma$ and $-\gamma$; hence, the matrix B differs. This property, named bidirectionality, has been thoroughly worked out in the excellent paper by McIsaac [2], where it turns out that even in the most general case of loss gyrotropic media there are classes of waveguide which exhibit this property.

In these cases, it will be possible to identify the adjoint solution with the eigenvector of the original waveguide belonging to $-\gamma$, i.e., the backward-running wave, but we are not allowed to conclude self-adjointness, as the authors obviously do by giving the condition [1, eqs. (25), (26)]

$$\boldsymbol{H}^a(x,y) = \boldsymbol{H}(x,y)$$

$$\boldsymbol{E}^a(x,y) = \boldsymbol{E}(x,y)$$

for the adjoint solution. This does not hold because the adjoint solution is the backward-running wave in the original waveguide whose fields are different from those of the wave running in the $+z$ direction with $+\gamma$.

Having drawn wrong conclusions about the adjoint fields, the authors neglect the terms with the factor γ in [1, eq. (35)]. However, these terms will not cancel, taking into account the correct adjoint solution. Thus, the final variational formula [1, eq. (37)] is wrong. No doubt it is a stationary formula, but not for solutions of the correct differential equation including the γ terms

$$\nabla_T \times \epsilon^{-1} \nabla_T \times \boldsymbol{H} + \gamma\left(\boldsymbol{u}_z \times \epsilon^{-1} \nabla_T \times \boldsymbol{H} + \nabla_T \times \epsilon^{-1} \boldsymbol{u}_z \times \boldsymbol{H}\right) + \gamma^2 \cdot \boldsymbol{u}_z \times \epsilon^{-1} \boldsymbol{u}_z \times \boldsymbol{H} - \omega^2 \epsilon_0 \mu_0 \boldsymbol{H} = 0 \qquad (1)$$

which is different from the Euler equation [1, eq. (41)] of the variational formula.

Thus, this formula will not give good approximations for the propagation constant γ by substituting trial functions for the magnetic field, nor will it give correct solutions for the magnetic field applying the Ritz procedure to the stationary formula.

Looking for reasons for the authors error, it is observed initially that they do not take into account the information given in [9] of their reference list ([4] here), where in (53) the backward-running wave has been identified as the adjoint solution, as well as in eq. (18) of their reference [10] (reference [5] here). Next, it is to be seen that they shift between three-dimensional and two-dimensional field problems in their considerations. Indeed, this can be done, but utmost care has to be taken because the properties of the corresponding operators may differ. So, while it is self-adjoint for the three-dimensional problem with a complex symmetric tensor, it is non-self-adjoint for the corresponding two-dimensional waveguide problem [3]. On the other hand, they do not try to derive the adjoint operator systematically by use of [1, eq. (19)], which will always give the correct result, commencing from the correct two-dimensional wave equation.

These properties of non-self-adjoint operators are not original. They are included in a thorough study of the electromagnetic variational principle [3]. This method has the advantage that it starts with physical reality, i.e., considering isotropic/gyrotropic, lossless/lossy media. The operators describing the physical problems are studied in detail. Their properties for three-dimensional as well as two-dimensional problems are derived for both Hermitian (complex) and symmetric (real) inner products. As one result among many, it has been found that for the problem at hand no self-adjoint formulation with symmetric (real) inner product is possible. It turns out that the only way to obtain a "variational

Manuscript received April 18, 1986.
The author is with Philips Kommunikations Industrie AG, Cologne, Germany.
IEEE Log Number 8610279.

[1] S. R. Cvetkovic and J. B. Davies, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-34, pp. 129–134, Jan. 1986.